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Measurement uncertainty in vibromonitoring systems and diagnostics reliability evaluation

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Abstract

This paper analyses the influence of measurement uncertainty to the decision-making model, where possible values are distributed normally and are limited by predefined limit values. While analysing evaluation of uncertainty in vibration monitoring systems, the latter due to the features and nature of the measurement process are distinguished into two main types: permanently installed systems and periodical vibration monitoring systems. The uncertainty model for these two different monitoring systems is also provided.

1. Introduction

The measurements of vibration as a process and its analysis are one of the main procedures in experimental investigations and applied research. Further solved problems, tasks and the progress of various processes might be impacted by the thorough implementation of this procedure. Years ago, as the measurement error, only the precision of measurement tools were mentioned, but now this concept do not involve all factors, which contribute to the measurement uncertainty [1]. In the latter decade the measurement results are related with uncertainty that is with the variance of the measurement results.

Although the ISO document "Guide to the expression of uncertainty in measurement" [2] provides general means and tools for uncertainty evaluation, still these means usually are used in the calibration procedure, as they indicate constant environmental conditions, otherwise when the measurable object is working under operating conditions these cases must be analyzed more thoroughly [3].

This paper shows that vibration measurements are dependent on the measurement purposes and procedures, as they have different measurement uncertainty and the latter may influence the result of the rotating machinery vibroacoustical diagnostics. The measurements, performed for other purposes would have the same effect. For example, in case of active vibration protection, the precision of control algorithm might be discovered, ensuring efficiency of operational parameters of vibroshock system, etc.

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In this stage, the methodology of vibration measurement uncertainty evaluation is applied for rotor systems vibration monitoring and diagnostic problems. The vibration monitoring and diagnostic systems usually collect a lot of data using several transducers or the transducer makes measurements at several machine points. This paper analyses the differences between these two measurement procedures. The decision made is based on these measurements and it is affected by measurement uncertainty.

2. Calculating measurement uncertainty in vibromonitoring systems

For any reliable measurement, it is important to evaluate expanded uncertainty U in compliance with the previously mentioned ISO Guide to the expression of uncertainty in measurement (GUM) [2].

The short summary for uncertainty evaluation is presented below. An estimate of the measurand Y, denoted by y is obtained using input estimates $x_1, x_2, \ldots, x_N$ through a functional relationship f

$$y = f(x_1, x_2, \ldots, x_N).$$

- Every effect, which significantly influences the measurement result, must be evaluated.
- Then each uncertainty component, which significantly contributes to the uncertainty of measurement, is described by a standard deviation u_i (standard uncertainty).
- Determine the combined standard uncertainty u_c, by combining the individual standard uncertainties (and covariances) according to [2,4]:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u^2(x_i, x_j)}. \quad (1)$$

  Here $u(x_i, x_j)$ is estimated covariance associated with x_i and x_j.
- Determine the expanded uncertainty U by multiplying u_c by a coverage factor k:

$$U = ku_c. \quad (2)$$

  In general, the value of coverage factor k is chosen on the basis of desired level of confidence to be associated with the uncertainty interval. A value of k is in the range 2–3. When the normal distribution applies and the uncertainty evaluate is reliable, usually $k = 2$ [2].
- When the result of the measurement y is reported, if k is not equal to 2 that must be stated.
  In order to calculate measurement uncertainty for vibration monitoring system, its structure also has to be analyzed [1,5].

The ISO standard ISO 13373-1 "Condition monitoring and diagnostics of machines—vibration condition monitoring—part 1: general procedures" distinguishes three types of monitoring systems:

- Permanently installed systems.
- Semi-permanent systems.
- Portable monitoring systems.

The typical structures of the permanently installed systems and portable monitoring systems [6,7] are shown in Fig. 1.

The parameters, affecting the vibration measurement process can be related to three main impacts: environmental impact (humidity, temperature), instrumental impact (calibration uncertainty), time component (measurements are performed in time intervals).

The uncertainty model for a permanently installed monitoring system was analyzed and the model was composed:

$$U_{\text{perm}} = k\sqrt{u_{\text{stat}}^2 + u_{\text{mat}}^2 + u_{\hat{K}_a}^2 + u_{K_{\text{ss}}}^2 + u_T^2 + u_H^2 + 2u_T u_H r(T, H)}. \quad (3)$$

Here u_{stat} is a statistical contribution, or random error, calculated from the data $\sigma/\sqrt{n}$, u_{mat} the contribution due to amplitude–frequency characteristic unevenness, $u_{\hat{K}_a}$ the error of transducer transformation coefficient,

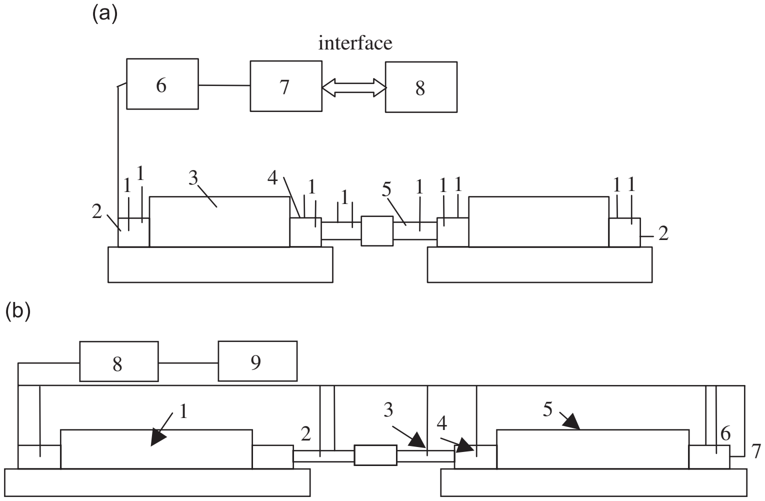

Fig. 1. Schemes of typical vibration monitoring systems. (a) Typical portable monitoring system: 1—radial measurement points, 2—axial measurement points, 3—driver, 4—phase reference, 5—driven process machinery, 6—accelerometer, 7—vibration measurement and analysis device, 8—PC. (b) Typical permanently installed monitoring system 1—driver; 2—shaft displacement probes (typical), 3—phase reference, 4—transducers on stationary bearing structure (typical), 5—driven process machine; 6—radial measurement points; 7—axial measurement points; 8—arrangement device; 9—PC.

$u_{K_{ss}}$ the error of transverse transformation coefficient, u_T the contribution of temperature T, u_H the contribution of humidity H, $r(T, H)$ the correlation between temperature and humidity.

The uncertainty model of portable vibration monitoring system could be calculated according to the formula:

$$U_{\text{port}} = k\sqrt{u_{\text{stat}}^2 + u_{\text{mat}}^2 + u_{\hat{K}_a}^2 + u_{K_{ss}}^2 + u_T^2 + u_H^2 + u_{\text{op}}^2}, \tag{4}$$

here u_{op} is error due to operator, as the measurements usually are performed manually by operator.

The main difference between stationary (permanently installed) and portable vibromonitoring systems is that here the main roles are played by different error types. In stationary system the measurements are performed constantly and the statistical uncertainty is very small, as the number of N is large, but the transducers are calibrated periodically, for example, once per year. Influence factors, determined during calibration, still affect the measurement procedure if it is performed in environmental conditions other than of calibration [3]. So the systematic error here plays the main role rather than the random. In case of periodic monitoring system, the transducer calibration is usually performed before every measurement set. But in contrary, as the measurements are rarer, their variance is bigger and the volume of data set is very small—the random error dominates in uncertainty value.

3. Impact of measurement uncertainty to diagnostics

The monitoring system observes the state of the object and collects information in order to make decision about further actions to be taken. Situation, when the decision must be made, comparing the measurement result with the predefined limit value, is depicted in Fig. 2. There are four possible cases: (A) when the measurement result, including uncertainty interval does not exceed the predefined limit, then the object is in the right condition; (B) when the measurement result does not exceed the predefined limit, but the upper limit of the uncertainty interval exceeds the limit; (C) When the measurement result exceeds the predefined limit value, but the lower uncertainty interval limit value still does not exceed the limit value; (D) the measurement value and the uncertainty interval exceeds the predefined limit value. In (B), (C) cases the uncertainty interval have influence on decision making about the object state.

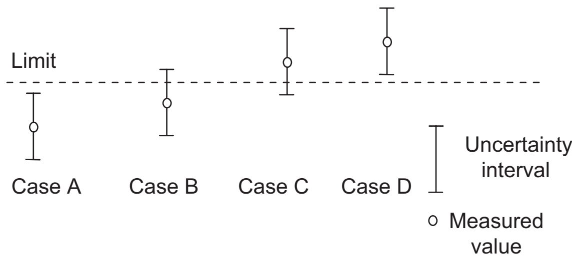

Fig. 2. Measurement uncertainty in the case of simple diagnostics.

In order to analyze the impact of uncertainty to the diagnostics result, the risk functions were investigated. Diagnosis is made according such simple rule:

- if the mean of the result $\bar{x}$ does not exceed the predefined limit x_0, then the decision D_1 is made;
- if the mean of the result $\bar{x}$ exceeds the predefined limit x_0, then the decision D_1 is made;
- the risk to make decision which does not correspond to a real situation is described by [8]

$$P(H_{21}) = P(D_1)P(x > x_0/D_1) = P_1 \int_{x_0}^{\infty} g(x/D_1)\,\mathrm{d}x, \tag{5}$$

$$P(H_{12}) = P(D_2)P(x < x_0/D_2) = P_2 \int_{-\infty}^{x_0} g(x/D_2)\,\mathrm{d}x. \tag{6}$$

Here, the $P(H_{21})$ describes the probability of making false decision that system is working not correctly and should be stopped; $P(H_{12})$ describes probability of false decision that system is working correctly, while the stop decision is made; D_1 is the correct state of the object; D_2 is incorrect state of the object; $g(x/D_i)$ is distribution of the set, when the object is in the state D_i; x_0 is predefined limit value to determine the state of the object. P_1, P_2 are predefined constant probabilities of making diagnosis D_1 and diagnosis $D_{2.}$

The measurement uncertainty affects only a part of Eqs. (5) or (6) which is under the integral sign, as the probabilities P_1 and P_2 are predefined and constant. As a rule, the data sets in monitoring systems are usually large, so the distribution of the monitoring system data set $g(x/D_i)$ might be described as

$$g(x/D_i) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(x-\bar{x}_1)^2/2\sigma^2}. \tag{7}$$

Here $\bar{x}$ is the mean value of the data set, further is also denoted as m; σ the standard deviation of the data set.

Possible uncertainty influence to decision-making probability is depicted in Fig. 3. The area of the first distribution, which falls outside the x_0 line, is the probability to make a false decision. When the uncertainty estimate U is added to the measurement result, the graph moves to the right. So, the area outside the x_0 line changes—it becomes bigger, in other words, the probability of making false decision increases. The same principles can be applied to the second curve in solid line; therefore we will analyze only the first case

$$\begin{aligned} f(\bar{x}, \sigma, U, x_0) &= g\big(x/D_1|_{x_0}\big) - g\big((x+U)/D_1|_{x_0}\big) \\ &= \frac{1}{\sigma\sqrt{2\pi}} \left(\int_{x_0}^{3\sigma} \mathrm{e}^{-(x-(\bar{x}+U))^2/2\sigma^2}\,\mathrm{d}x - \int_{x_0}^{3\sigma} \mathrm{e}^{-(x-\bar{x})^2/2\sigma^2}\,\mathrm{d}x \right). \end{aligned} \tag{8}$$

Here the function $f(\bar{x}, \sigma, x_0, U)$ describes the change of false decision probability when the area is constrained by limit value x_0 and 3σ. The function will depend on the mean, variance, limit value and uncertainty. Fig. 4 demonstrates the dependence of the function on the sample mean and uncertainty. The limit value was chosen as being in the middle between the mean and right uncertainty added. The variance was chosen big enough to form a right profile of distribution. So, the bigger is mean value, the character of decision-making probability becomes more nonlinear while considering uncertainty value.

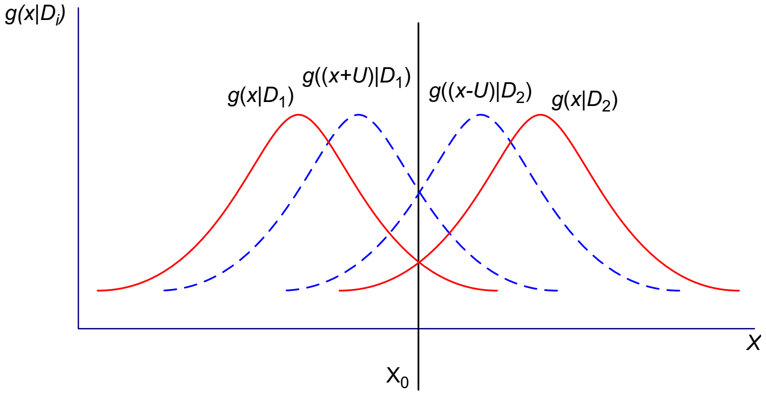

Fig. 3. Uncertainty influence to the decision-making distribution.

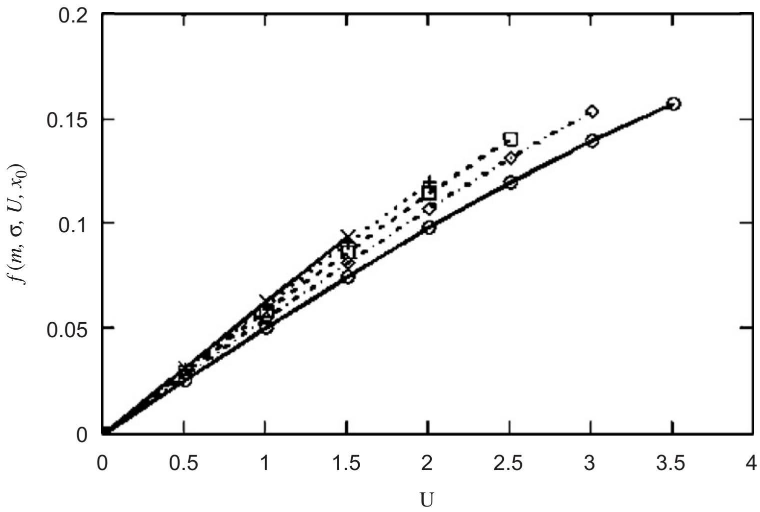

Fig. 4. Dependence of decision-making probability to uncertainty and the sample mean when limit value is changing: —$f(m_1, \sigma, U_1, x_{01})$; + · + —$f(m_2, \sigma, U_2, x_{02})$; —$f(m_3, \sigma, U_3, x_{03})$; —$f(m_4, \sigma, U_4, x_{04})$; —$f(m_5, \sigma, U_5, x_{05})$.

The Fig. 5 demonstrates dependence of the function on the uncertainty and sample mean. The limit value and variance was not changed. The graph demonstrates that decision-making probability change depends on the uncertainty while the mean changing, but it has a nonlinear character.

As the change of variance does not have influence to decision-making probability change—all graphs had almost the same shape and meanings, the ratio of the mean and uncertainty was analysed (Fig. 6).

The maximum uncertainty was chosen to be half of the mean value. The figure demonstrates that there is a clear dependence on this ratio—the bigger is uncertainty, the bigger fail probability we can expect. We suggest using this ratio as a parameter to calculate a decision-making probability change. For example, when we meet the case *B* demonstrated in the Fig. 2, it is hard to decide what decision should be made—should we accept this as a good state, or reject as the bad? Calculating this ratio would be easier to evaluate a significance of uncertainty.

Applying this to different vibration monitoring systems, we should notice that in stationary vibration monitoring systems the variance of the normal distribution will be small enough due to large data sets, so in critical cases, the uncertainty influence will show the effect immediately. In the case of portable monitoring system, the data sets are small and variance is much bigger. Due to that, the uncertainty might have a big

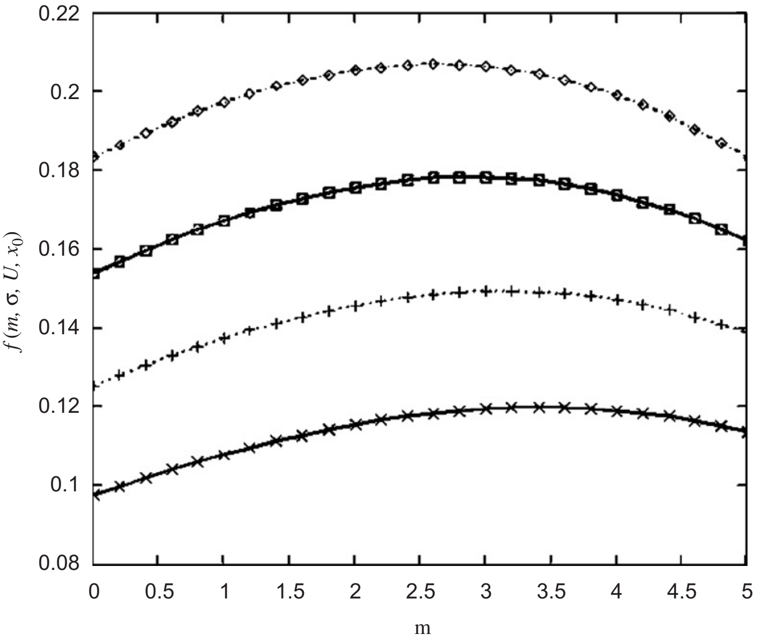

Fig. 5. Dependence of decision-making probability to sample mean and uncertainty when limit value is not changing: ✕—✕— $f(m,\sigma,U_1,x_0)$; + · + —$f(m,\sigma,U_2,x_0)$; ⊖—⊖—$f(m,\sigma,U_3,x_0)$; ▪◇▪—$f(m,\sigma,U_4,x_0)$.

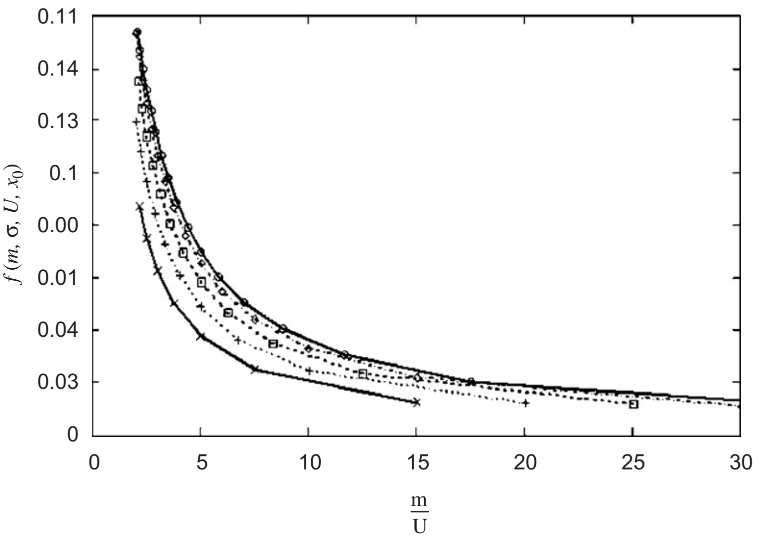

Fig. 6. Dependence of decision-making probability to uncertainty and the ratio of the mean to uncertainty: ✕—✕—$f(m_1,\sigma,U_1,x_{01})$; + · + —$f(m_2,\sigma,U_2,x_{02})$; ⊡ ⊡—$f(m_3,\sigma,U_3,x_{03})$; ▪◇▪—$f(m_4,\sigma,U_4,x_{04})$; ⊖—⊖—$f(m_5,\sigma,U_5,x_{05})$.

influence to decision making with portable systems having initial data. The data set might be expanded using the methodology described in Ref. [9], while transformation function will enlarge data set, in this way reducing the statistical uncertainty component. Using this method additional uncertainty contribution should be added, which calculates the impact of the increase of the set to measurement reliability. In the other hand, this contribution has a less impact than the difference between the primal statistical uncertainties and uncertainties calculated after the transformation.

4. Conclusions

- Assuming, that measurement uncertainty integrates of two characteristic components: systematic and random, measurement uncertainty of portable and permanently installed vibration monitoring systems have different character. In the case of stationary vibration monitoring system, the systematic character of measurement uncertainty emphasizes. In case of portable (periodic) monitoring system—random character of uncertainty emphasizes.
- In monitoring systems, if simple decision rules are applied, it is necessary to provide the uncertainty estimate.
- The ratio of the data set mean and the calculated uncertainty may be used as a parameter to evaluate the possible impact of uncertainty to decision making.

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